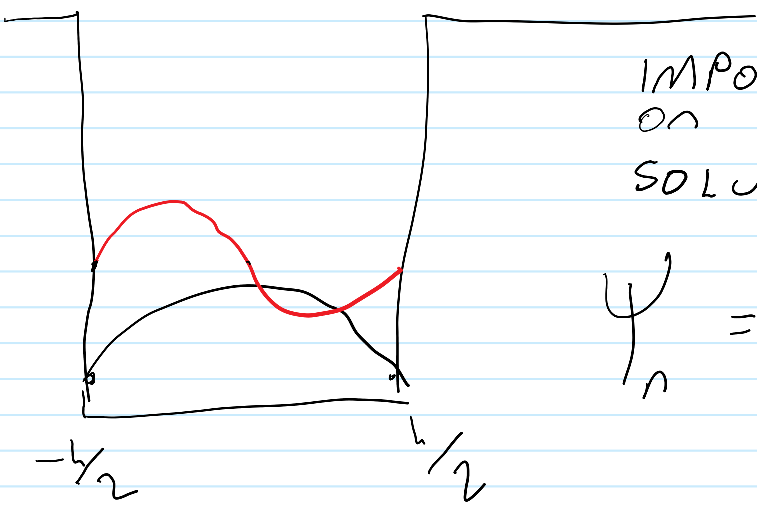
Ch. 14.4

Infinite square well with boundaries at -L/2 and +L/2 instead of 0 to L.

a) IF cos(kx) are solutions, what must k be
b) If sin(kx) are solutions, what must k's be
c) Allowed energies.

The solutions in the well must appear the same --have the same looking wavefunctions, and same energies. We have just shifted what we mean by coordinates.



Impose same B.C. on new suggested solutions

$\psi_n = \sin k_n x = 0$

at $x = \pm L/2$

True if

$k_n \frac{L}{2} = \frac{n\pi}{2}$

$n = 2, 4, 6, 8 \ldots$

same k's at $-L/2$ also

$\cos k_n x = 0$ at $x = \pm L/2$

True if $k_n \frac{L}{2} = \frac{n\pi}{2}$ $n = 1, 3, 5 \ldots$

Numerically the values of k's are all the same as before (same wavelengths). They must be the same. However, since the well is shifted over by so many parts of a cycle, some of the previous solutions are shifted from sin's to cos's solutions.

The energies relate to k's in the same way, so we get the same energies for all n's.

$$E_n = \frac{n^2 h^2}{8 m L^2}$$

14.6 For the infinite square well in state n=1 (ground) and state n=2 (first excited), what is the probability of finding a particle between L/4 and 3L/4

$$\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

$$P_1\left(\tfrac{L}{4} \to \tfrac{3L}{4}\right) = \frac{2}{L}\int_{L/4}^{3L/4} \sin^2\left(\frac{1\pi x}{L}\right) dx$$

$$\int \sin^2 ax\, dx = \frac{1}{2}x - \frac{1}{4a}\sin^2(2ax)$$

$$P_1 = 0.818$$

$$P_2 = 0.5$$

4.9 A neutron is confined (so infinite square well) to a box with L=3.5E-15m. Find wavefunctions and energies for states n=1,2.

$$m = 1.675 \times 10^{-27}\ kg$$

$$L = 3.5 \times 10^{-15}\ m$$ (about a nucleus)

$$E_n = \frac{n^2 h^2}{8 m L^2}$$

$$\psi_1 = \sqrt{\frac{2}{L}} \sin\left(\frac{1 \pi x}{L}\right) = \frac{2.39 \times 10^7}{\sqrt{m}} \sin\left(8.98 \times 10^{14} x\right)$$

$$\psi_2 = \frac{2.39 \times 10^7}{\sqrt{m}} \sin\left(1.795 \times \frac{10^{15}}{m} \times x\right)$$

$$E_1 = 2.675 \times 10^{-12}\ J$$

$$E_2 = 4 E_1$$

14.12 It takes 4.00eV to excite an electron in a one dimensional infinite square well from n=1 to n=2. What does this tell you about the well?

$$E_2 - E_1 = \frac{4 h^2}{8 m L^2} - \frac{1 h^2}{8 m L^2}$$

$$\begin{array}{l} 4.0 \text{ eV} \\ 6.4 \times 10^{-19} \text{ J} \end{array} = \frac{3}{8} \frac{h^2}{m L^2}$$

$$L = 5.31 \times 10^{-10} \text{ m}$$

14.13	An electron is in a well with L=3.78nm, What is wavelength of n=1 to n=2 transition?

from 14.12 $\quad \Delta E = \frac{3}{8} \frac{h^2}{m L^2}$

$$= 1.265 \times 10^{-20} \text{ J}$$

$$= 0.079 \text{ eV}$$

$$c = \nu \lambda \qquad h\nu = \Delta E$$

$$\lambda = \frac{hc}{\Delta E} = 1.57 \times 10^{-5} \text{ m}$$

$$= c h \frac{8}{3} \frac{m L^2}{h^2}$$

So, increasing L will increase the wavelength (and lower the energy).

14.16 For a particle in an infinite square well, with the bottom of the well at V=0---the energy is all kinetic. So with E=p^2/2m show that p can be written as nh/(2L).

$$\frac{n^2 h^2}{8 m L^2} = \frac{p^2}{2m} \rightarrow p = n \frac{h}{2L}$$

Then either from picture of solutions, or sin(kx) ......noting k=2π/λ from the nature of periodic functions. Boundary conditions required L was filled with a whole number of half wavelengths.

half cycle

$$kL = n\pi$$

$$k_n = \frac{2\pi}{\lambda} \qquad \underbrace{p = \hbar k}_{\frac{nh}{2L}} = \frac{\hbar 2\pi}{\lambda} = \frac{h}{\lambda}$$

So, L is filled with n half waves

$$\lambda_n = \frac{2L}{n}$$

14.19 For infinite square well, show that Ax+B solution is for E =0 case. Note--inside well, V=0.

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + 0\,\psi = E\,\psi$$

subst $\quad \psi = Ax + B$

$\quad \psi'' = 0$

so $\quad 0 = E * (Ax + B)$

only if $\quad E = 0$

Wavefunctions with no curvature have infinite wavelength---so energy of zero. Also, have no way to meet B.C. in this case with Ax+B.

b) $\quad \psi(x=0) = B = 0$ B.C. (given ψ)

$\quad \psi(x=L) = AL + 0 = 0$ B.C.

so $A = 0$

$$\underline{\psi = 0}$$